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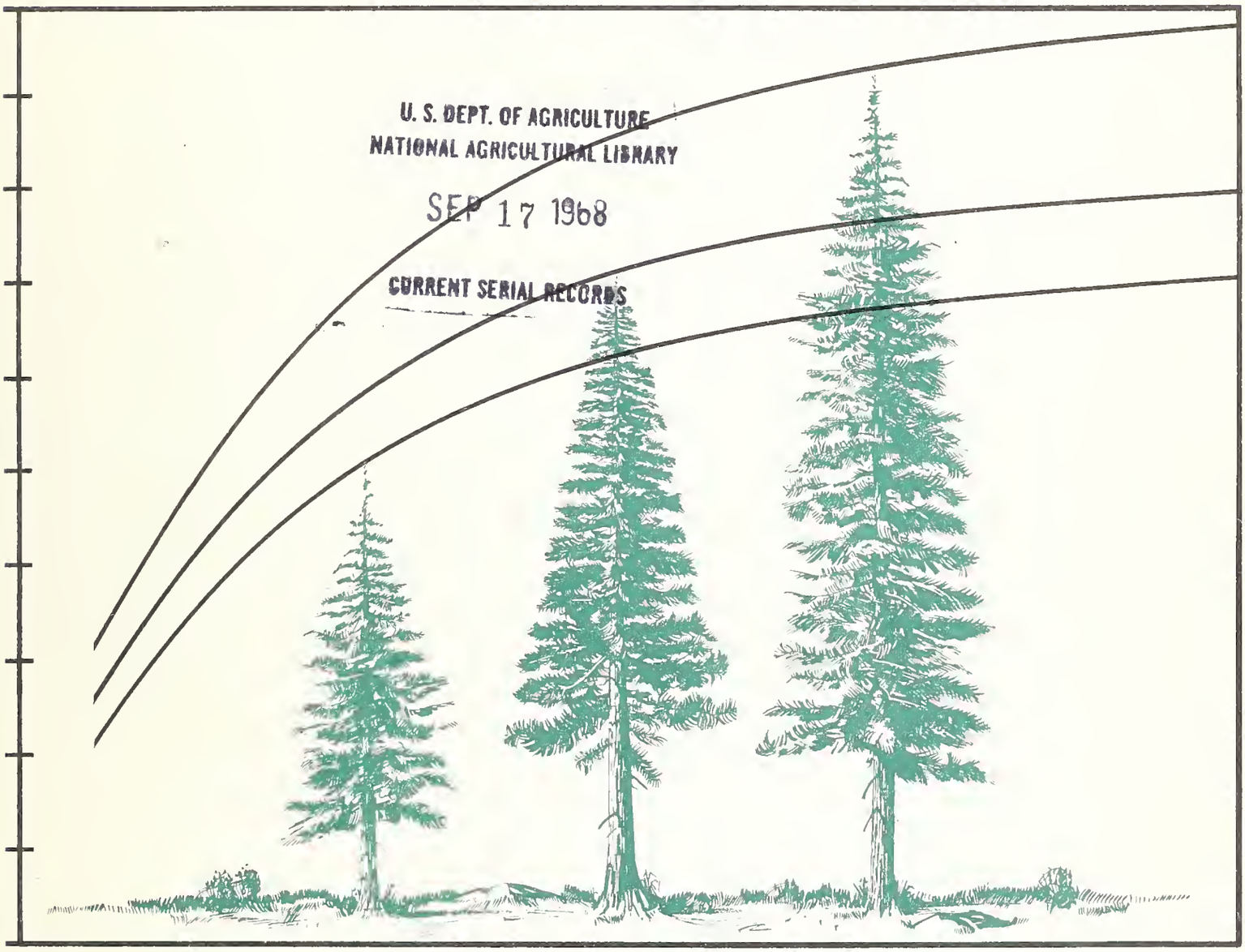
A METHOD FOR CONSTRUCTING SITE INDEX CURVES FROM MEASUREMENTS OF TREE AGE AND HEIGHT-ITS APPLICATION TO INLAND DOUGLAS-FIR

by JAMES E. BRICKELL



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U.S. Forest Service Research Paper INT-47, 1968
Intermountain Forest and Range Experiment Station
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U.S. Forest Service
Research Paper INT-47
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A METHOD FOR CONSTRUCTING SITE INDEX CURVES FROM MEASUREMENTS OF TREE AGE AND HEIGHT--ITS APPLICATION TO INLAND DOUGLAS-FIR

by

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FOREWORD

This paper describes a method of constructing site index curves from one-time measurements of age and height of trees in the dominant stand. Site index curves for Engelmann spruce (Picea engelmannii Parry) have already been constructed using this method (1).[1] In this paper curves are constructed for inland Douglas-fir (Pseudotsuga menziesii var. glauca (Mirb.) Franco) to illustrate how to use the method.

This method has the advantage of being applicable to the kind of site tree data usually taken in the course of forest inventories. Moreover, the disadvantages of these data are compensated for to some degree by the fact that the resulting site index curves will usually be used with the same kind of data, containing the same hidden effects of past damage.

The method is a generalization of the Osborne-Schumacher method (10) which assumes equal representation of a particular site quality in all age classes. The generalization does not require residuals about the mean curve of height over age to be normally distributed. A disadvantage of the method proposed in this paper is that it requires a relatively large number of sample trees--at least 1,500 paired measurements of age and height should be available.

The site index curves for inland Douglas-fir resulting from using this method are recommended for use in the northern and central Rocky Mountain area. These curves are shown on a page inserted in the middle of the publication. This page is designed so it can be detached for use in the field. Table 3 on page 14 shows height of trees in the dominant stand for various combinations of age and site index. Equations by which site index can be estimated from age and height are given on pages 20 and 22. These equations are especially useful in adapting the system of site index curves to use in electronic computer programs.

[1] Numbers in parentheses refer to Literature Cited.

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A METHOD FOR CONSTRUCTING SITE INDEX CURVES FROM MEASUREMENTS OF TREE AGE AND HEIGHT--ITS APPLICATION TO INLAND DOUGLAS-FIR

INTRODUCTION

For many years the most widely followed practice in describing forest site quality has been the use of a quantitative measuring system--site index curves. These must be developed for individual species and are supposed to represent the relationship between height and age of the average dominant and codominant trees on the site. These curves can be derived from measurements that indicate height and age of selected trees at different periods in time, or from measurements of height and age at one point in time on a large number of trees that range in age from young to old.

The first method, which uses either stem analysis data or recurrent measurements of selected trees on permanent sample plots, has usually been followed in recent years when site index curves or growth and yield information are the objectives of the study and data are collected for that purpose. Trees are selected for stem analysis or recurrent measurements so as to exclude those that have been subjected to height-growth-inhibiting injuries. Reliable use of curves derived from this kind of data presupposes that trees measured for comparison with the curves also have been free of injuries throughout their lives. When selecting trees to compare with the curves in the course of normal fieldwork it is seldom practical or even possible to limit a sample to the kind of uninjured trees on which the curves were based. Too often a tree may have sustained some injury in the past that cannot be detected by an observer on the ground. As a result, some trees that have suffered height-growth-inhibiting injuries are generally included in samples. Such sample trees will give an underestimate of site quality when compared with site index curves based on uninjured trees.

This is not intended as a criticism of the use of stem analysis or recurrent measurement data for site index curve development when such data are obtained with probability sampling techniques. On the other hand, when one is forced to use one-time measurements of tree height and age, such as may be collected in forest inventories, there is reason to think that both the trees on which the curves are based and those measured subsequently to estimate site index are from similar populations; that is, populations of trees as they exist in the forest. Thus, the curves are developed from the same kind of measurements as those with which they will be used, which may tend to reduce any bias arising from an observer's inability to detect the effects of previous top damage.

The site index curves currently being used for many species in the Western United States are based on paired measurements of age and height taken on trees in the dominant stand at one point in time. Such data are more easily collected in the course of forest inventories and are, for some species, the only data presently available. This is likely to be true, to an extent, for some time in the future. It is the purpose of this paper to show how this kind of data can be used to construct site index curves which are free of bias that could be introduced by the assumptions implicit in older methods.

METHODS FOR CONSTRUCTING CURVES

The methods of constructing site index curves as described in the forestry literature will result in either anamorphic (proportional) curves, or polymorphic (nonproportional) curves. Polymorphic curves vary in shape from one site to another. If their shape is well determined, they are generally considered more consistent with the known growth habits of trees. Therefore, polymorphic curves are to be preferred for estimating site index (6, 12, 2).

The Osborne-Schumacher method results in polymorphic curves based on the standard deviation of residuals about the mean curve of height over age. Fractions and multiples of the standard deviation are used in the construction of the actual curves at various site index levels. The assumption on which this method is based was stated by Osborne and Schumacher as follows:

> A necessary condition for the correct determination of site index is that it be intrinsically independent of plot age; for then the proportion of the sample plots whose site indexes are better--or poorer--than any given site index is the same, in the long run, at one age as at another; and, conversely, site index may be identified by associating it with the probability of occurrence. For example, the site qualities that are the most productive 10 percent of the area in a timber type should be represented by 10 percent of the sample plots in each age class whose height of average dominant--or dominant and codominant tree is greatest.

The procedure for site index curve construction given in this paper rests on the validity of the Osborne-Schumacher assumption, which can be related to site index curves as follows: If all land of a given site quality within a forest region is represented by areas of land in all age classes proportional to that part of the total land area in that particular site quality,[2] then the average height growth curve of trees in the dominant stand on land of that site quality can be represented by a curve that is above or below the mean curve of height on age by a distance which represents the same probability level in the distribution of residuals at all ages. This distance will, of course, vary with age.

In certain cases patterns of land use in the past may render this assumption invalid. Curtis (3) found this to be so with some tree species in New England. In this study it has been assumed that previous land use patterns have not operated to produce such an effect for the following reasons:

1. The data were drawn from a wide geographic area embracing several different economic regions

2. Most of the data came from National Forests where cutting practices have been rather different from those on smaller private holdings and

3. The effects of cutting practices and land use, if any, have been felt for at least 2 centuries less time in the Mountain States than in New England.

In the absence of evidence to the contrary, it is implicitly assumed that there is no relation between site and the incidence of fire, windfall, and insect damage.

PRELIMINARY ANALYSIS OF DATA

Unfortunately, only total age was recorded for the trees measured in this study, although age at breast height is a more reliable age upon which to base site index curves (9, 13, 5). Before a tree reaches this height, competition from brush, weeds, grass, etc., is likely to influence height growth to such an extent that any effect which the site itself may have on height growth is entirely obscured.

---

[2] That is, the expected value of

$$a_{ij} / \sum^{j} a_{ij} = \sum^{i} a_{ij} / \sum\sum^{ij} a_{ij} ,$$

where a_{ij} denotes the land area in the i^{th} age class and the j^{th} site class. $\sum^{j} a_{ij}$ is all land bearing stands in the i^{th} age class. $\sum^{i} a_{ij}$ is all land in the j^{th} site class, and $\sum\sum^{ij} a_{ij}$ is total land area for the species in the forest region being considered.

More than 3,000 Douglas-fir trees were measured for total height as well as total age. All were located on forest inventory plots in the northern and central Rocky Mountains (fig. 1). From one to three trees were measured at each location and each tree was counted as a separate observation. To qualify for sampling, trees had to be in the dominant portion of the stand and appear to have been free of injury and suppression throughout their lives.

The overall area shown in figure 1 was divided into five subregions, differing in respect to general climatic conditions that might influence tree growth (14). Each site tree observation was tagged by a subregion code of 1 through 5. This allowed later segregation of the site index predictions so as to test the shape of the final system of curves within each of the five subregions.

Trees 200 years of age and older were rejected in constructing the curves, leaving 3,183 trees in the sample. Trees 200 years of age or older are likely to be so large in diameter that increment borers ordinarily available would not reach to the pith; therefore, an accurate estimate of age would not be obtained. Also, top damage that the tree may have sustained in the past is often difficult to detect in an old tree.

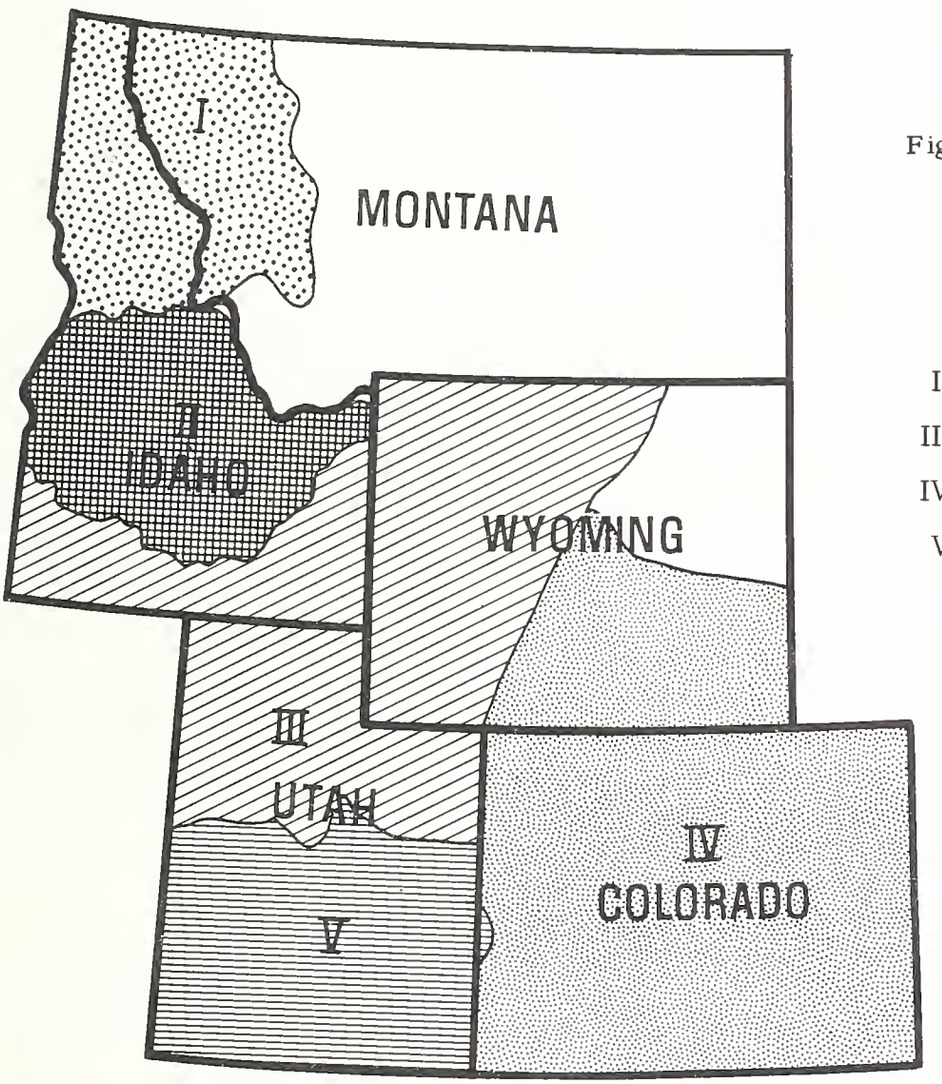


Figure 1.--The area from which the data used in this study were drawn. Subregions are:

I
II
III
IV
V

The trees left in the sample were sorted into 20-year-age groups and the distribution of heights within age groups was plotted. It was apparent that the distributions of heights within most of the 20-year-age groups were skewed, possibly due to the spread of ages within classes, and the intraclass correlation of height with age. It seemed likely, in any case, that the distribution of residuals about the mean curve of height over age would be nonnormal. If so, then the Osborne-Schumacher method could not be used unless skewness and kurtosis were the same at all ages. Otherwise, one standard deviation above the mean curve would reflect a different level of probability at different points along the curve, and any resulting site curves would be inconsistent with the basic assumption as stated on page 2.

DERIVATION OF CURVES

The method used in estimating the curves presented in this paper follows five distinct steps:

1. Sorting the site tree observations according to ascending values of total tree age and dividing the sample into agewise segments, each segment containing about 100 trees.

2. Fitting an average curve of total height over total age to the mean age and mean height of sample segments.

3. Locating a series of probability percentage points in the distribution of residuals around the average curve within each segment of the sample.

4. Fitting a series of curves of the same algebraic form to all points located at the same probability level in the several sample segments. Each curve is then associated with a particular level of probability of occurrence.

5. Harmonizing the series of curves obtained in step 4 above by expressing the coefficients of the curve equations as functions of site index. This results in one equation for the system of site index curves.

SORTING SITE TREE OBSERVATIONS

Because of the evident skewness of the distributions of heights within 20-year-age classes, the site tree observations were separated into groups, or sample segments, according to ascending values of total tree age. These sample segments were established so as to include 100 observations each, as nearly as was possible without breaking up groups of trees of the same age. Statistical tests for significance of kurtosis require samples of this size if sufficient sensitivity is to be attained.

A total of 29 classes was thus obtained, each class representing a varying age span--most of these fell within the range of 4 to 7 years. The number of trees in each sample segment, mean age, mean height, and standard deviation of the residuals around the average curve of height over age are shown in table 1.

Table 1.--Agewise segmenting of site tree observations for the purpose of residual distribution analysis

Sample segment	Age span	No. of trees	Mean age	Mean height	S[1]
	Years		Years	Feet	
1	0-35	99	28.4	30.1	11.5
2	36-43	105	40.1	41.4	13.5
3	44-48	98	46.1	43.5	13.6
4	49-53	107	50.9	50.4	16.0
5	54-58	116	56.1	54.4	17.5
6	59-62	117	60.3	57.1	16.1
7	63-66	118	64.6	60.2	17.2
8	67-70	134	68.7	61.3	18.7
9	71-74	108	72.3	59.9	18.8
10	75-78	105	76.4	61.2	19.9
11	79-82	116	80.5	63.2	19.8
12	83-86	99	84.6	61.5	17.5
13	87-91	108	89.3	65.2	21.4
14	92-96	106	94.1	66.4	18.9
15	97-101	107	99.1	72.3	20.4
16	102-107	108	104.8	65.5	19.5
17	108-112	102	110.0	72.0	18.7
18	113-118	105	115.5	71.6	20.6
19	119-122	101	120.3	71.2	19.7
20	123-128	105	125.5	73.2	21.3
21	129-134	111	131.3	75.6	21.0
22	135-140	137	137.7	76.6	22.8
23	141-147	109	144.3	70.9	20.3
24	148-153	106	150.4	78.3	22.5
25	154-160	107	157.9	71.2	21.4
26	161-168	103	164.7	75.9	22.0
27	169-175	104	172.6	74.8	20.1
28	176-184	103	180.1	76.0	24.2
29	185-199	139	190.8	76.2	21.8

[1]Standard deviation of height residuals around the mean curve of height over age.

FITTING AVERAGE HEIGHT-AGE CURVE

To estimate the mean curve of height on age, a growth function of suitable form was fitted to the sample segment means of age and height, and properly weighted according to the number of trees in each segment. Several growth equations were fitted to the data and evaluated in an attempt to express the average relationship between tree height and tree age. The most appropriate was found to be:

$$H = a\,[1 - b \cdot \exp(-kA)]^{1/1-m}$$

where:

H = total tree height

A = total tree age

a, b, k, m are coefficients of the equation to be estimated by the method of least squares.

exp(-kA) means e, the base of Naperian logarithms, taken to the (-kA) power.

This model and its use as a growth curve have been discussed in detail by Richards (12). It can be seen that the function is not linear in its parameters, nor can it be made so by transformation. To fit this equation, it is necessary to use iterative numerical methods that, in this case, were those embodied in a computer program produced by the Esso Research and Engineering Corporation. The least squares criterion was satisfied by the following values of the coefficients:

a = 76.0

b = 1.0

k = 0.0290833

m = 0.398601

The estimates of the first two coefficients that were used to start the iterative fitting process remain unchanged. After the second two coefficients were estimated, any reduction in the error sum of squares obtained by changing coefficients a and b was insignificant because the initial estimates were so close to the optimum least squares solution. This average height over age curve and the points to which it was fitted are shown in figure 2.

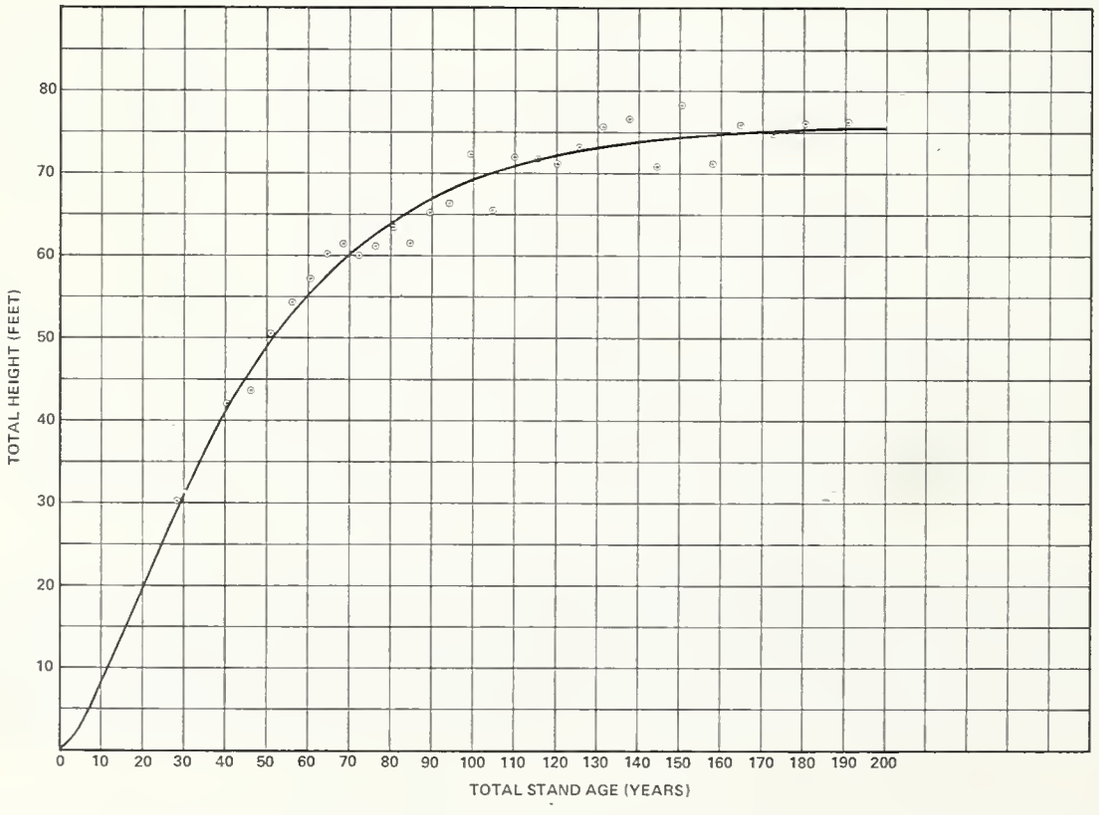


Figure 2.--The average curve of height over age for inland Douglas-fir. The plotted points are (mean age-mean height) coordinates for each sample segment.

LOCATING PROBABILITY PERCENTAGE POINTS

After a prediction equation for average height was obtained, a height was predicted for each tree in the sample. This predicted height was subtracted from the tree's actual height, and the residuals obtained thereby were accumulated separately for each of the 29 sample segments. The 100 or so residuals in each sample segment, then, formed a frequency distribution. These distributions were nonnormal in most cases.

For each within-segment distribution of residuals the standard deviation (S) was calculated. Estimates of the moment ratios β_1 and β_2 were computed as indices of skewness and kurtosis according to the method of k-statistics given by Kendall and Stuart (8) where the k-statistics are sample estimates of the distribution cumulants. The moment ratios were computed from the corresponding cumulant ratios.

Figures 3 and 4 show the coefficients β_1 and β_2, respectively, plotted over corresponding sample segment mean age. It will be noted that nearly two-thirds of the segment distributions are significantly skewed, while only six depart significantly from the mesokurtic condition, three being platykurtic and three leptokurtic. Considering both measures of nonnormality, only eight of the 29 distributions within sample segments can be called normal distributions.

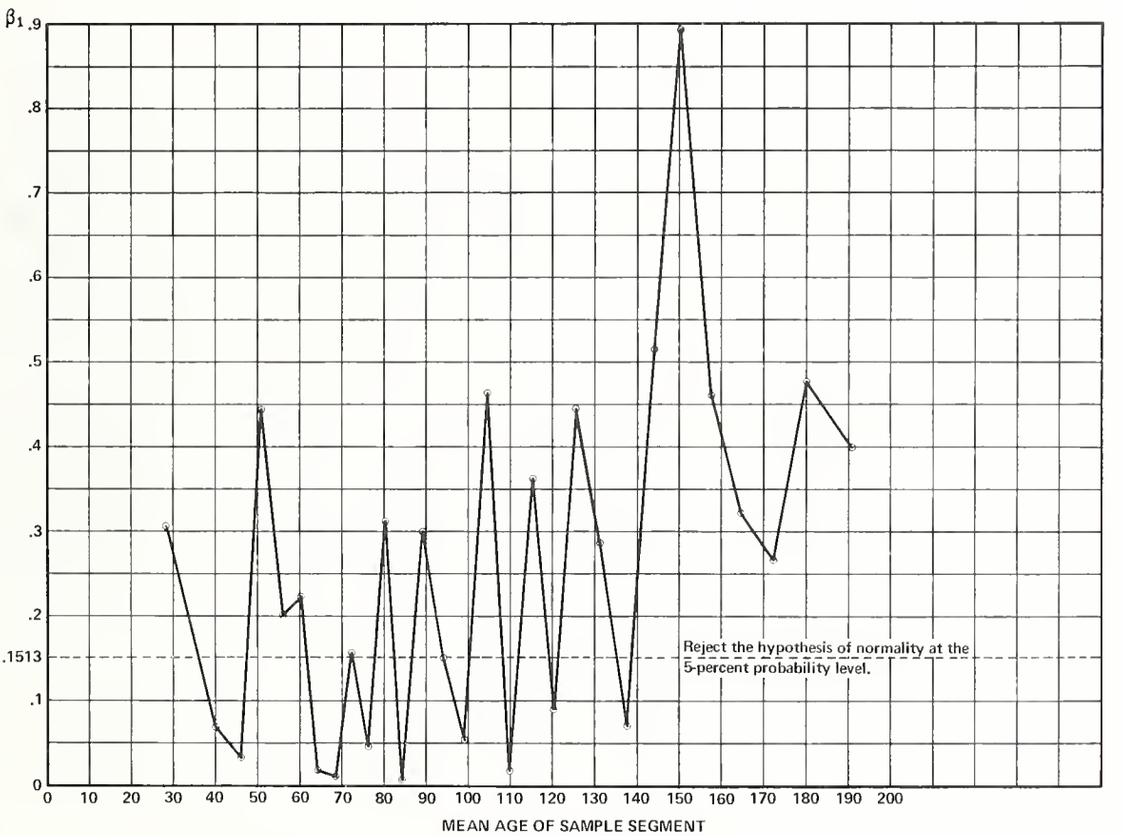


Figure 3.--Estimates of the β_1 coefficient plotted over corresponding sample segment mean age.

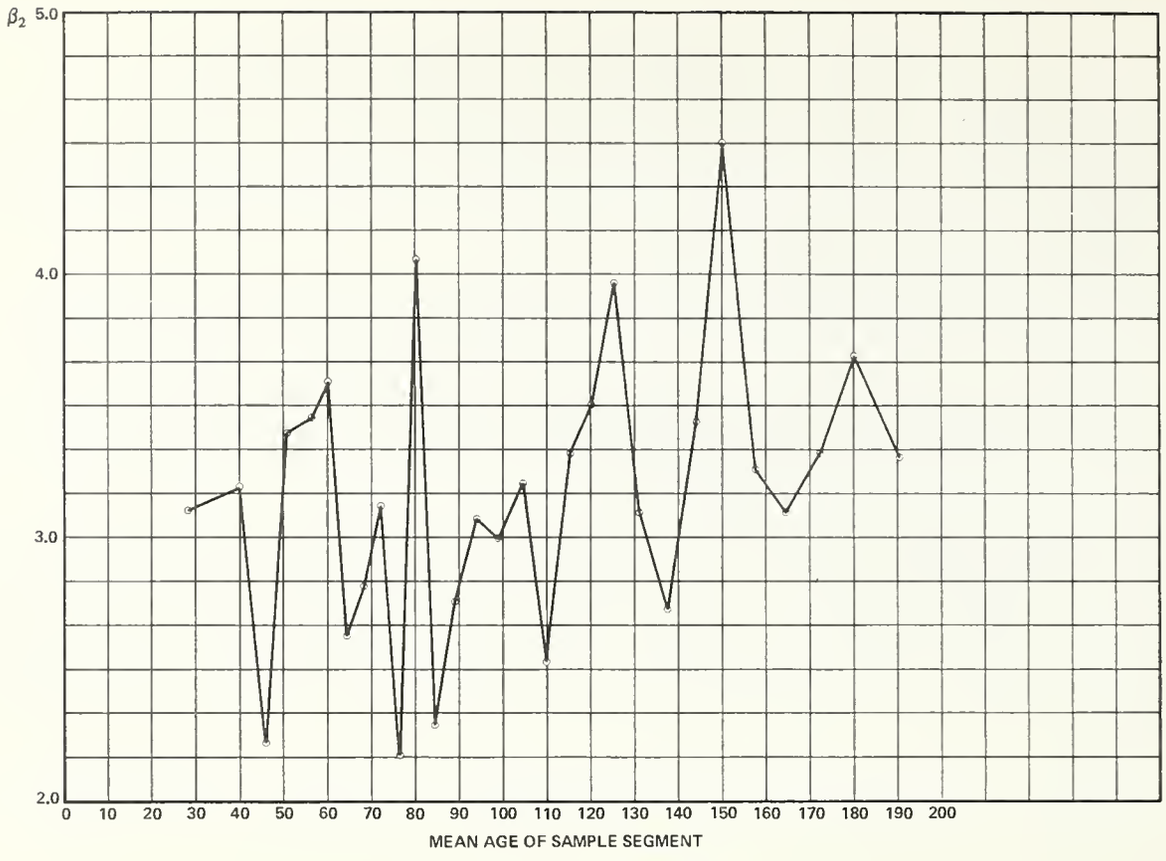
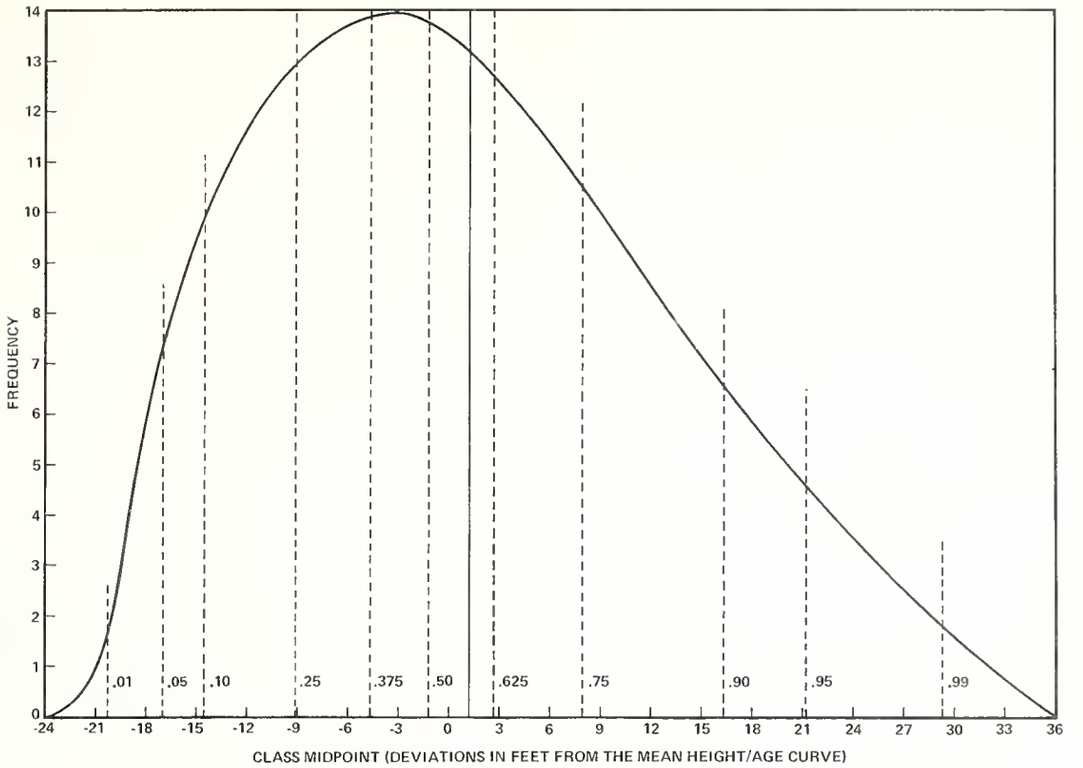


Figure 4.--Estimates of the β_2 coefficient plotted over corresponding sample segment mean age.

Definite trends of both β_1 and β_2 with age can be seen in figures 3 and 4. Because of these trends, the probability attached to a point one standard deviation from the mean in each distribution will vary with age. Consequently, the Osborne-Schumacher method could not be used without modification. In a series of skewed distributions such as this, the mean will not be located at the same point of probability in each distribution. The median, however, is by definition the 50-percent probability point in any distribution; if the distribution is symmetrical, the mean and the median coincide. Because the distributions of residuals in this case are not symmetrical, the mean curve of height on age does not represent points of equal probability at all ages.

Therefore, under the previously mentioned assumptions of equal representation of site quality in all age classes, the mean curve does not represent a true height growth curve. Rather, a curve fitted to the medians, or 50-percent probability points, of the within-segment distributions should describe a true height growth curve at that particular level of site index. Further, this curve would divide the population of forest sites into two parts, half being of poorer site quality, and half being better.

Figure 5 shows a skewed frequency distribution typical of the distributions of residuals within sample segments. The points marked .01, .05, .10, .25, .75, .90, etc., show the points in the distribution below which 1 percent, 5 percent, 75 percent, etc., of the forest sites in the population are expected to fall. All of the distributions of residuals in the 29 sample segments could be adequately described by Pearson's system of frequency curves (4). All except one of the distributions are of Type I or Type IV (fig. 6).



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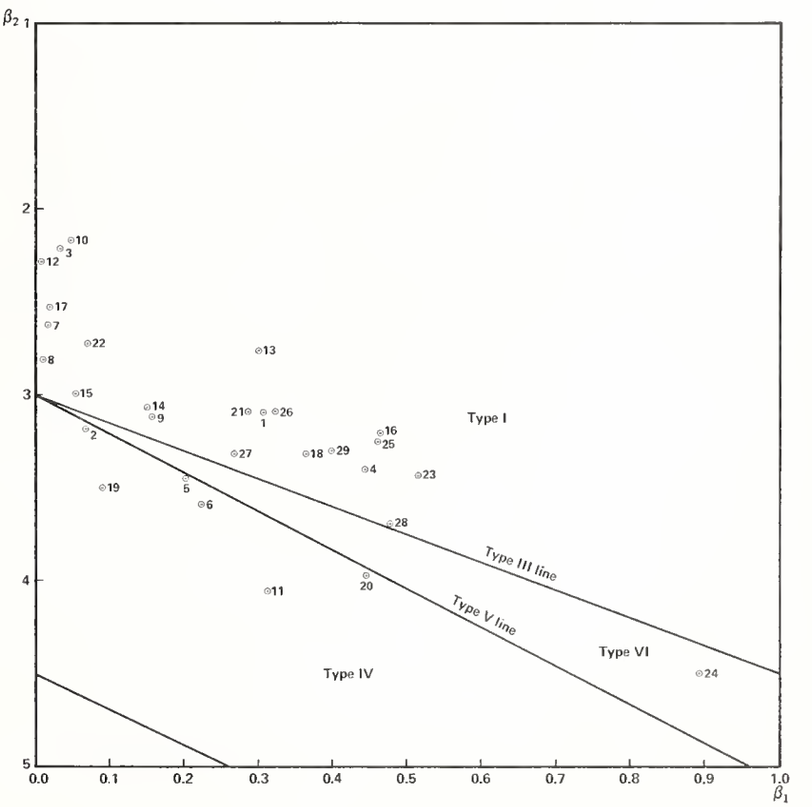

Figure 6.--Showing the types, according to Pearson's system, of the 29 within-segment distributions of residuals. Number beside each plotted point identifies the sample segment from which it came.

Nine percentage points of the distribution of heights within each age group were estimated using the tables of probability percentage points for Pearson's frequency distributions (7). These points represent the heights such that 1, 5, 10, 25, 50, 75, 90, 95, and 99 percent of the trees in each sample segment were shorter than the particular limit. The relative locations of these percentage points are shown in figure 5.

Heights at the several different probability levels were computed in the following manner for each sample segment:

1. For a given probability level, the distance from the mean in standard units[3] was found from the probability tables (7). Call this distance α.

2. Multiply α by S, which is the standard deviation of residuals within the sample segments.

3. A height corresponding to the mean age of the sample segment was predicted using the equation for the mean height over age curve (fig. 3). Call this predicted mean height $\hat{\bar{H}}$.

4. Call the mean residual of the within-segment distribution Δ.[4]

5. Then the height associated with this particular probability level (p) in the i^{th} sample segment is:

$$H_{ip} = \hat{\bar{H}} + \Delta + \alpha \cdot S .$$

In addition to the nine heights determined in this way, heights were obtained for the 37.5 and the 62.5 probability percentage points from the third and fifth octiles of the residual frequency distribution within each sample segment. This means of estimation is somewhat less precise than the procedure followed for the other nine height estimates. Nevertheless, it is unbiased.

FITTING CURVES TO EACH SERIES OF PROBABILITY PERCENTAGE POINTS

For all sample segments, the heights associated with a particular probability level were plotted over segment mean age. The curve drawn through the plotted points was either above or below the median curve, depending upon its probability level. It was not necessarily proportional to the average curve; in fact it was not proportional for any of the 11 probability levels. Yet no mathematical artifacts prohibited proportionality if the data had so indicated. Curves derived in this manner for the 10-, 50-, and 90-percent probability levels are shown in figure 7. To avoid confusion, curves for the other probability levels are not shown, but their appearance is quite similar. Nevertheless, the 11 curves differ in shape enough to indicate polymorphism that should be preserved in the final system of curves.

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[3] This means that the distribution's standard deviation is 1.0. Thus, a distance from the average of 1.5 or 2.3 means 1.5 or 2.3 times as far from the average as the standard deviation.

[4] Recall that the mean curve of height on age was fitted to the entire distribution of tree ages and heights. Therefore, there is no reason to expect that the mean residual within a sample segment will be zero.

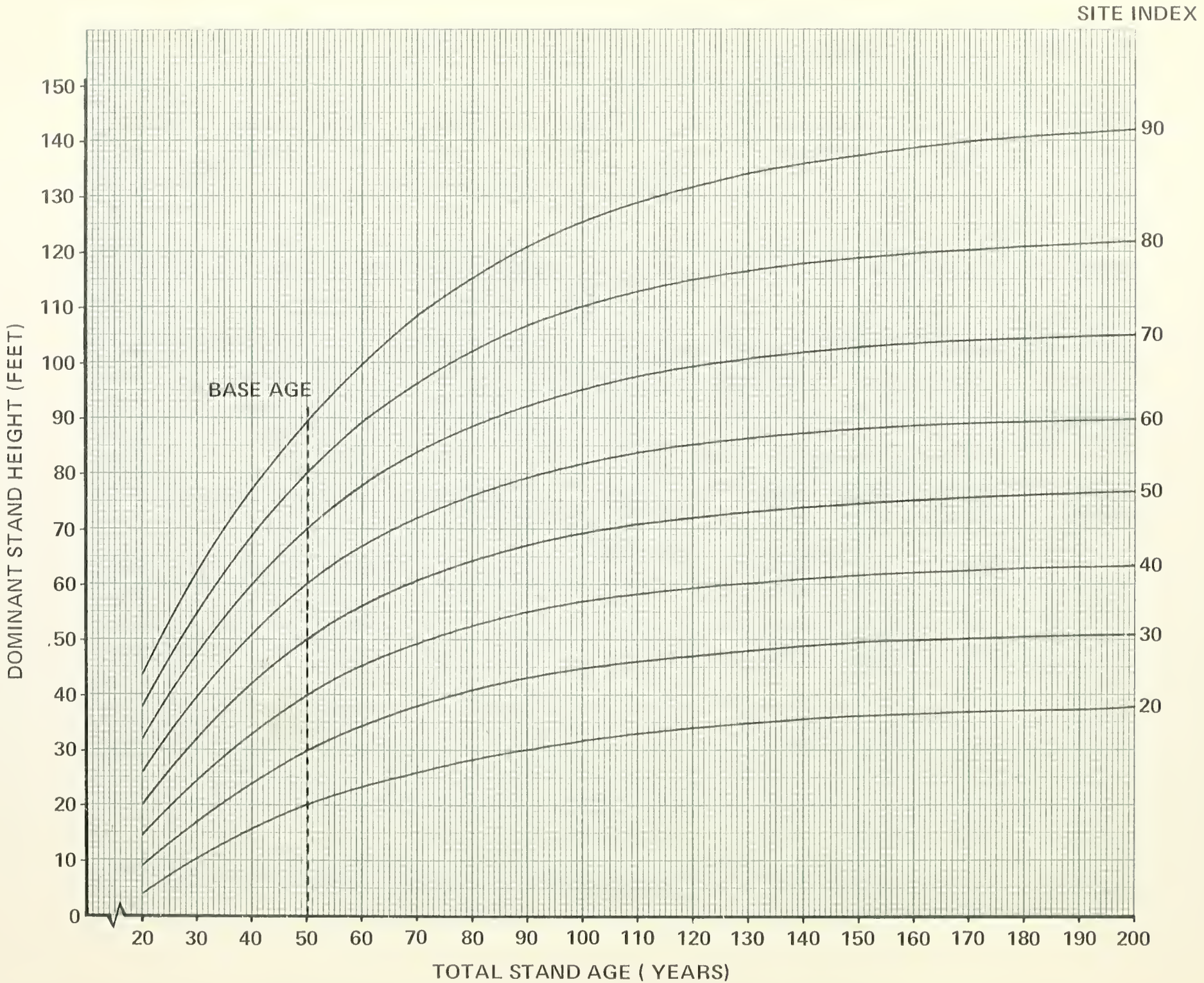
SITE INDEX CURVES FOR INLAND DOUGLAS-FIR IN THE NORTHERN AND CENTRAL ROCKY MOUNTAINS

HOW TO USE SITE INDEX CURVES

1. To estimate the average site index for a stand of inland Douglas-fir, measure total age and total height of trees that are in the dominant portion of the stand. Site trees should appear to have been in the dominant stand throughout their lives without having suffered top damage that inhibited height growth. The number of sample trees necessary to give any stated precision of estimate will depend on the variance in site index between trees in the stand. This is influenced by the size of the stand and the degree of its heterogeneity in site quality.

2. Using the curves on the other side of this sheet, or figure 11 or table 3 in Research Paper INT-47 from which this sheet was taken, estimate a site index value for each sample tree. (For computer applications, either of the equations on pages 20-22 of INT-47 can be used.)

3. The average site index for all sample trees will be the best estimate of average site index for the area sampled if selection of site trees was random with respect to site quality.


SITE INDEX
DOMINANT STAND HEIGHT (FEET)
BASE AGE
TOTAL STAND AGE (YEARS)
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
20
30
40
50
60
70
80
90
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110
120
130
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150
160
170
180
190
200
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20

Brickell, James E.

1968. A method for constructing site index curves from measurements of tree age and height--its application to inland Douglas-fir. U.S. Dep. Agr., Forest Serv., Intermountain Forest and Range Exp. Sta., Ogden, Utah. 23 pp., illus. (U.S. Forest Serv. Res. Pap. INT-47).

Describes a method for constructing site index curves from paired measurements of tree age and tree height. Use of this method results in polymorphic curves, but anamorphic curves would be obtained if the data so indicate. Includes site index curves for inland Douglas-fir in the northern and central Rocky Mountains to illustrate application of method. A table and equations for use in computer programs also are given.

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1968. A method for constructing site index curves from measurements of tree age and height--its application to inland Douglas-fir. U.S. Dep. Agr., Forest Serv., Intermountain Forest and Range Exp. Sta., Ogden, Utah. 23 pp., illus. (U.S. Forest Serv. Res. Pap. INT-47).

Describes a method for constructing site index curves from paired measurements of tree age and tree height. Use of this method results in polymorphic curves, but anamorphic curves would be obtained if the data so indicate. Includes site index curves for inland Douglas-fir in the northern and central Rocky Mountains to illustrate application of method. A table and equations for use in computer programs also are given.

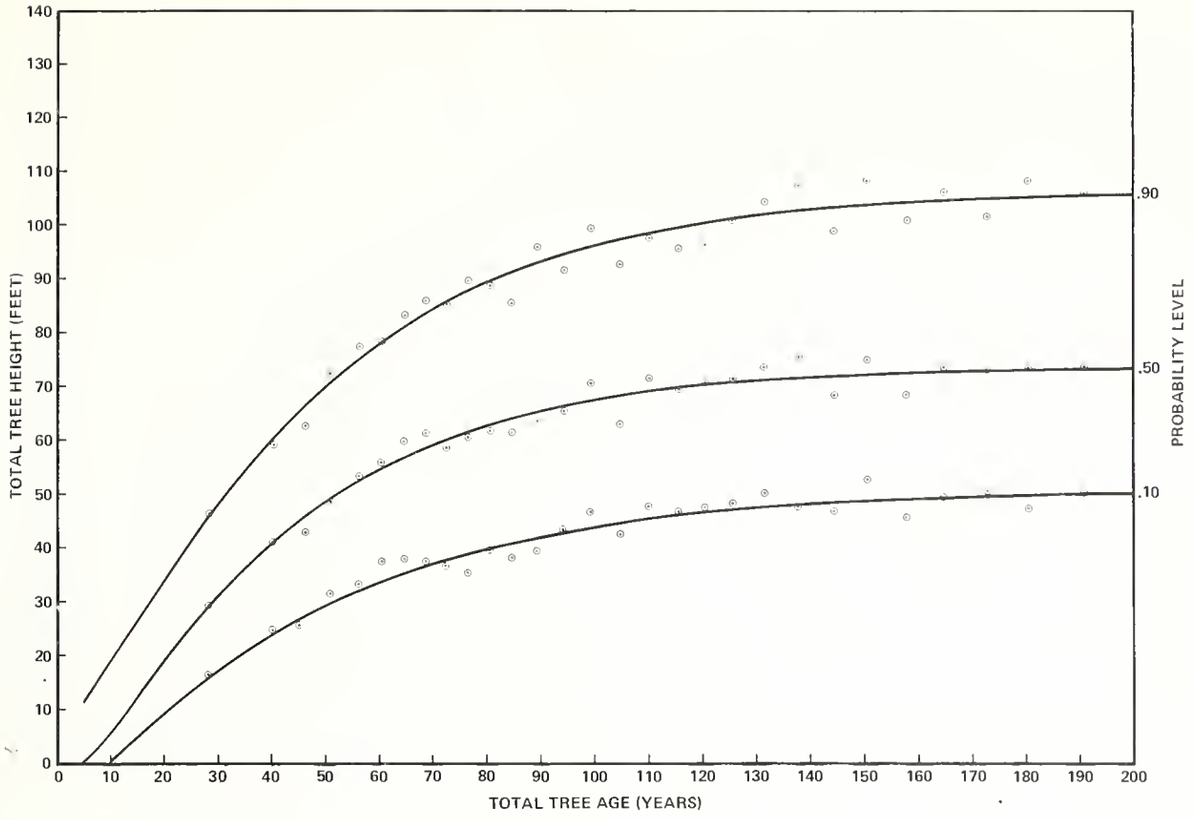


Figure 7.--Curves of height over age for the 10, 50, and 90 percentage of probability levels.

The growth function that was used for the mean height-age curve was fitted to each of the 11 equal-probability level curves. The estimates of the coefficients a, b, k, and m obtained for these curves are shown in table 2, where the different curve levels are identified by the percentage of trees expected to have a lower site index. Note that the value of the coefficient m remained at the value used to start the iterative fitting process in several cases. Evidently, this coefficient is quite poorly determined when there is as much variation around the curve as there was in these cases, and no observations exist in the neighborhood of the curve's inflection point. According to Richards (11), it is the m coefficient that determines the inflection point of the sigmoid curve described by this growth function.

Using the coefficients given, each curve was solved for height at an age of 50 years. This assigned a site index to each curve and to the set of points to which the curve was fitted. The root mean square residual of the points derived from probability percentages for each of the 11 curves follows:

Percent	Feet
0.01	2.48151
.05	1.97699
.10	2.10783
.25	2.41299
.375	2.68735
.50	2.39172
.625	2.97846
.75	2.49292
.90	2.91963
.95	3.42081
.99	5.86715

Table 2.--Coefficients and estimated site index of eleven polymorphic height over age curves

Probability level (percent)	Coefficients a	b	k	m	Site index (feet)
0.01	39.0055	0.940887	0.0194496	0.436483	17.87440
.05	45.5291	1.30145	.0229012	.1	25.13584
.10	50.6858	1.24789	.0230806	.1	29.07733
.25	60.3456	1.07016	.0270850	.308415	37.81088
.375	67.1239	1.0	.0290830	.398600	43.19466
.50	73.9369	1.16358	.0278033	.191505	48.42383
.625	80.9312	1.04128	.0308835	.398600	53.27936
.75	89.6517	.934803	.0290830	.398600	59.51743
.90	106.2990	.848792	.0267493	.398600	69.90275
.95	117.9460	.967460	.0232968	.175890	76.26919
.99	139.7280	.770341	.0209054	.323633	87.53819

HARMONIZING THE SYSTEM OF SITE INDEX CURVES

For each of the four growth function coefficients, the 11 different values shown in table 2 were plotted over the corresponding site index assigned to that curve, resulting in the scatter diagrams shown in figure 8. Apparently there was no trend of the coefficient m with site index. Coefficients a, b, and k, however, showed correlation with site index. The expressions best relating the coefficients to site index seemed to be:

$$f_1(S) = a = p_0 + p_1 S + p_2 S^2 \quad (2)$$

$$f_2(S) = b = q_0 + q_1 S \quad (3)$$

$$f_3(S) = k = \exp(r_0 + r_1 S + r_2 S^2) \text{ where} \quad (4)$$

S = site index.

Then, assigning a site index to each set of points bearing the same probability level, the function

$$H = f_1(S) \cdot \left\{1 - f_2(S) \cdot \exp\left[f_3(S) \cdot A\right]\right\}^{1/1-m}$$

was fitted to all 319 of the points from which the series of 11 curves had been derived. The fitting was done by the same iterative numerical process as was used for the less complex function. Each of the 319 points was assigned a weight proportional to the number of sample trees in the sample segment in which the point was located and inversely proportional to the residual sum of squares about the curve to which the point belonged. That is:

$W_{ij} \propto N_i / \Sigma res^2_j$, where

W_{ij} = the weight of the point located at the j^{th} probability level in the i^{th} sample segment.

N_i = the number of site tree observations in the i^{th} sample segment.

Σres^2_j = the sum of squared residuals about the curve at the j^{th} probability level.

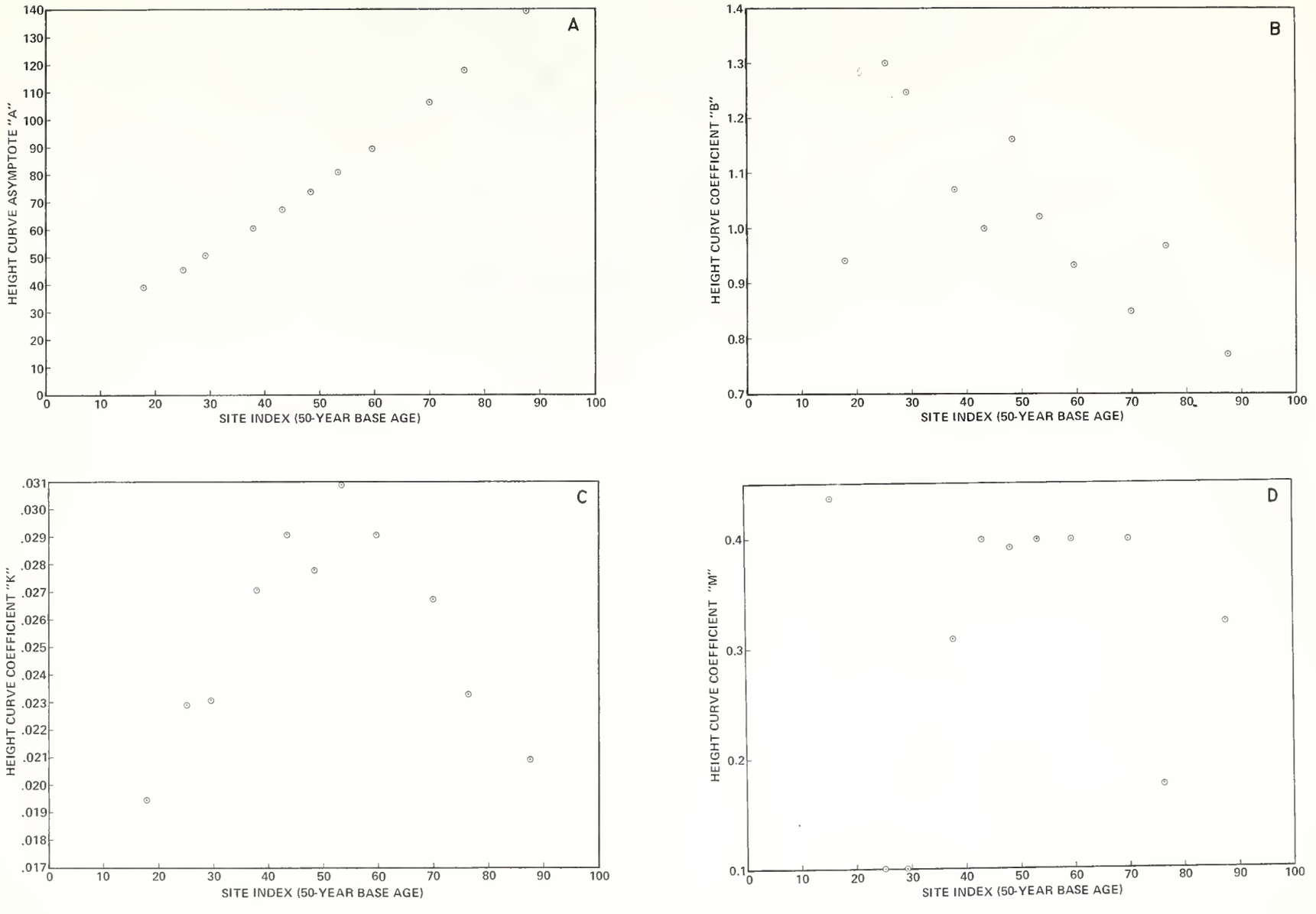


Figure 8.--A, The growth curve "a" coefficient plotted over site index; B, the growth curve "b" coefficient plotted over site index; C, the growth curve "k" coefficient plotted over site index; D, the growth curve "m" coefficient plotted over site index.

The equation finally obtained that expressed tree height as a function of age and site index is:

$$H = a\,[1-b\cdot\exp(-kA)]^{1/1-m}, \text{ where} \quad (5)$$

$$a = 20.618 + 0.864711\cdot S + 0.00529738\cdot S^2 \quad (6)$$

$$b = 1.40694 - 0.0042396\cdot S \quad (7)$$

$$k = \exp(-4.14672 + 0.017648\cdot S - 0.000161345\cdot S^2) \quad (8)$$

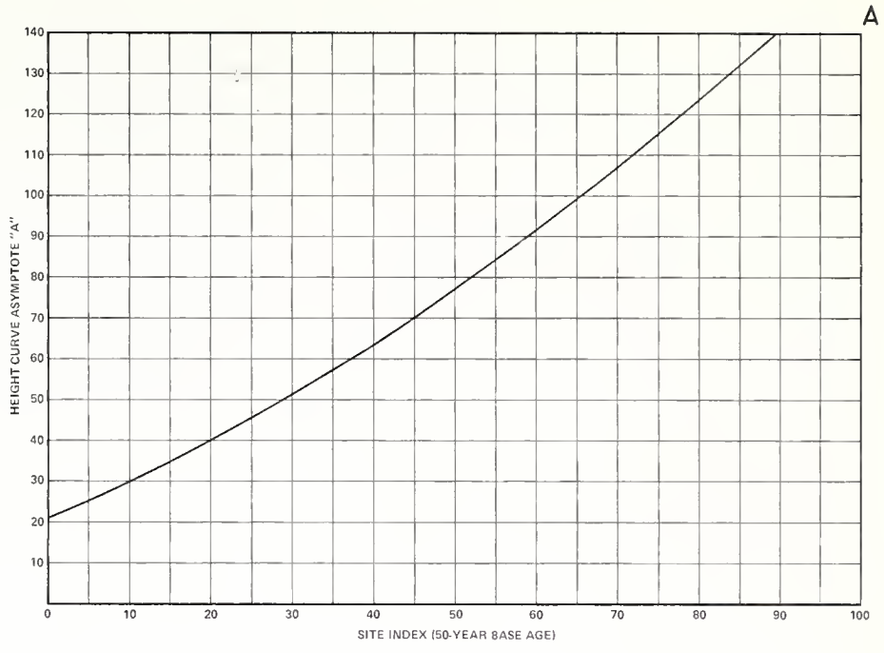
$$m = .05730421$$

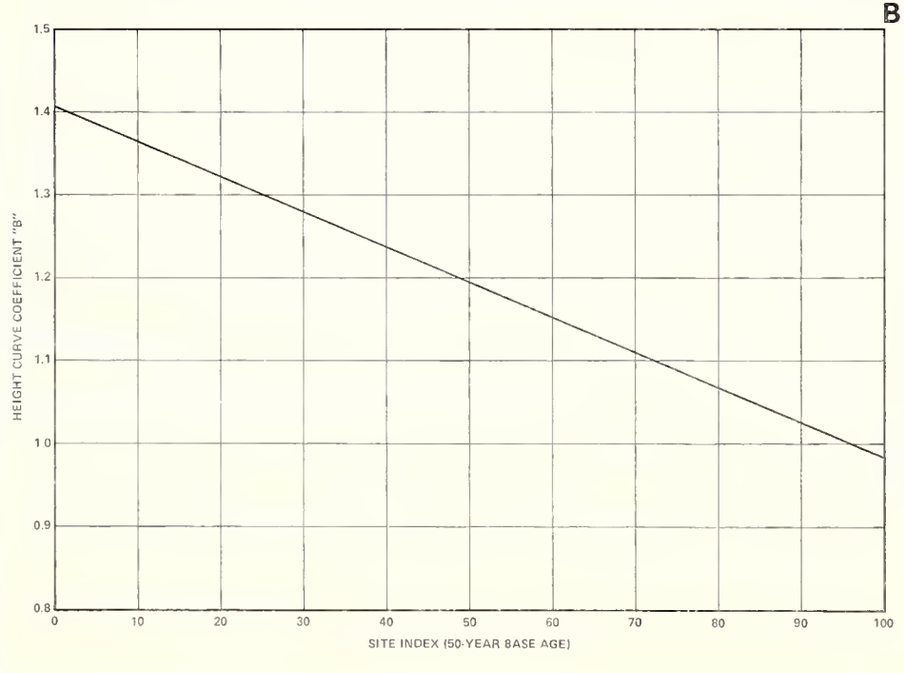
The relationships between coefficients a, b, and k, and site index, as they exist in the final equation, are shown in figure 9. The coefficient m remains the same for all levels of site index. Curves generated by this equation are shown on the heavy tearout sheet at the center of this paper. These are the completed site index curves. Table 3 gives height as estimated from these curves according to total tree age and site index.

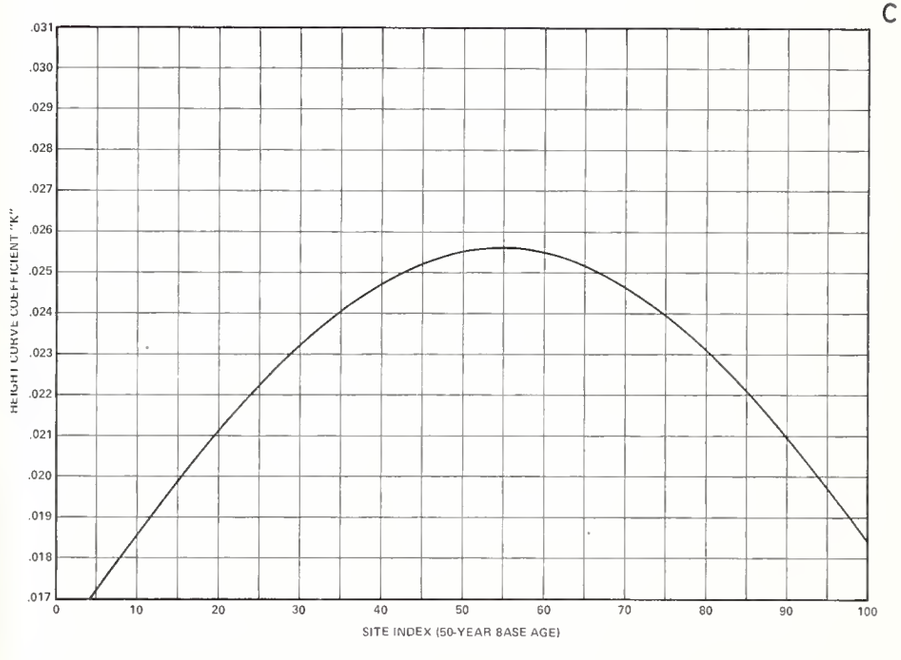
Table 3.--Height of trees in the dominant stand by age and site index

Total age (years)	Site index							
	20	30	40	50	60	70	80	90
	Feet							
20	4	9	14	20	26	32	38	44
30	11	18	25	32	40	48	55	62
40	16	24	33	42	51	60	69	78
50	20	30	40	50	60	70	80	90
60	23	34	45	56	67	78	89	100
70	26	38	49	61	72	84	96	108
80	29	41	53	64	76	89	102	115
90	30	43	55	67	79	92	106	121
100	32	45	57	69	82	95	110	125
110	33	46	59	71	84	98	113	129
120	34	47	60	72	85	99	115	132
130	35	48	61	73	87	101	117	134
140	36	49	61	74	87	102	118	136
150	36	49	62	75	88	103	119	138
160	37	50	62	75	89	103	120	139
170	37	50	63	76	89	104	121	140
180	37	51	63	76	89	104	121	141
190	37	51	63	76	90	105	122	141
200	38	51	63	76	90	105	122	142

Figure 9.--A, The final relationship between the growth curve "a" coefficient and site index; B, the final relationship between the growth curve "b" coefficient and site index; C, the final relationship between the growth curve "k" coefficient and site index.







TESTING CURVES FOR INDEPENDENCE OF AGE AND SITE INDEX

Lacking additional data, only a test of consistency between subregions was possible. To do this, a site index based on the newly developed curves was assigned to each tree in the sample. Then the trees were separated into five groups according to the geographic subregion from which they were drawn.

TESTING WITHIN EACH SUBREGION

Within each areal group and within all groups combined, an attempt was made to find significant trends of estimated site index with tree age by fitting the equation:

$$S = b_0 + b_1A + b_2A^2 + b_3A^3 \qquad (9)$$

to the joint distribution of tree age (A) and estimated site index (S). The multiple correlation coefficients thus obtained for data from subregions I, III, IV, and V failed to indicate significant difference from zero at the 95-percent probability level. Therefore, the hypothesis that there is correlation between tree age and estimated site index was rejected for these subregions, and the two were assumed independent.

In the data from subregion II, however, it appeared that correlation between age and estimated site index did exist. The source of this correlation was further traced to the data from six National Forests in central Idaho. This implied that height-growth curves of inland Douglas-fir have the same shape for a given level of site index throughout the northern and central Rocky Mountains except in a relatively small area of central Idaho.

EFFECT OF OVERSTORY COMPETITION

Experience with Douglas-fir in central Idaho indicated that in at least part of the area this species is largely found in the understory.[5] Often it takes the form of a younger stand coming up beneath old-growth ponderosa pine. Competition from an overstory may have thus changed the shape of the typical growth curve somewhat.

Supporting this possibility was the fact that the correlation between age and estimated site index was present to the greatest extent in trees less than 70 years old. On the average, these trees were not as tall for their age as was expected. Competition may have reduced the height growth of young trees in some areas but, if so, then the reduction in diameter growth should have been even greater. It is common knowledge that diameter growth is more sensitive to competition than height growth. This made possible a statistical test to determine whether or not competition was more severe for young trees in the six National Forests studied than it was for young Douglas-fir trees throughout the rest of the northern Rocky Mountain area.

For the test, all trees in the sample 70 years old or less were separated into two groups according to the area from which they were drawn. Trees from the areas where competition was thought to have inhibited height growth comprised one set of data, and trees from all other areas comprised the other. To each set of data was fitted the equation:

$\ln H = b_0 + b_1 \ln D$, where

$\ln H$ = Naperian logarithm of total tree height

$\ln D$ = Naperian logarithm of d.b.h.

b_0, b_1 = regression coefficients

[5] Communication from Charles A. Wellner, Assistant Director, Intermountain Forest and Range Experiment Station.

The results are shown in figure 10. The two regression equations obtained are significantly different at the 95-percent confidence level. It can be seen that trees in central Idaho usually have a smaller d.b.h. for the same height. This indicates that trees measured as site trees were indeed subject to greater competition in parts of central Idaho than elsewhere. It does not mean that better site quality had produced more rapid height growth--hence better tree form--because these trees were shorter than expected. The site seems to be actually better than young trees would indicate.

If the foregoing can be accepted as the reason for lack of independence between age and estimated site index on six National Forests, then it can be assumed that, but for the pressure of overstory competition on these particular Forests, the height growth curves would be the same as they are elsewhere in the northern Rocky Mountains. This assumption was made, and the system of curves published here is recommended for use throughout the area.

The foregoing discussion might give rise to two questions: (a) what biasing influence could the inclusion of subregion II data have had on the system of site index curves so far as their use in other subregions is concerned? and (b) why were trees which had been affected by competition measured as site trees in the first place?

The answer to the first question is that although some biasing effect would seem inevitable, statistical tests failed to show that age and estimated site index were not independent. Therefore, any bias in the curves would be of little, if any, practical consequence. The second question can only be answered by pointing out that Forest Survey crews were obliged by field sampling rules to measure Douglas-fir site trees on or near the sample plot so long as the stand was predominantly Douglas-fir by basal area, even though the Douglas-fir may have been mostly in an understory position.

ESTIMATION EQUATIONS FOR USE IN COMPUTER PROGRAMS

The equation that expresses tree height as a function of age and site index cannot be readily solved for site index given age and height. The numerical procedure necessary to achieve a solution would probably not converge to a solution rapidly in many cases and would thus require an excessive amount of computer time. Table lookup procedures require the storage of a table in the computer memory core, which has definite disadvantages, including the necessity for interpolation between cells of the table. Interpolation in a nonlinear system can lead to inaccuracies.

For these reasons, it was decided to devise a means by which site index could be computed directly from age and height. To do this, a regression equation was developed that approximated the surface of site index over age and height. The surface was generated from the equation of height over age and site index (equation 5). Figure 11 shows site index plotted over height for several different age levels. At age 50, site index is identically equal to height, because 50 years is the index base age of the system of curves. It was found that equations of the form:

$$S = b_0 + b_1 H + b_2 H^2 + b_3 H^3 + b_4 H^5 \qquad (10)$$

gave the best fit for most ages, including age 50, where b_1 has to equal unity and all other coefficients are zero. Fitting this equation to the curves of site index over height for each

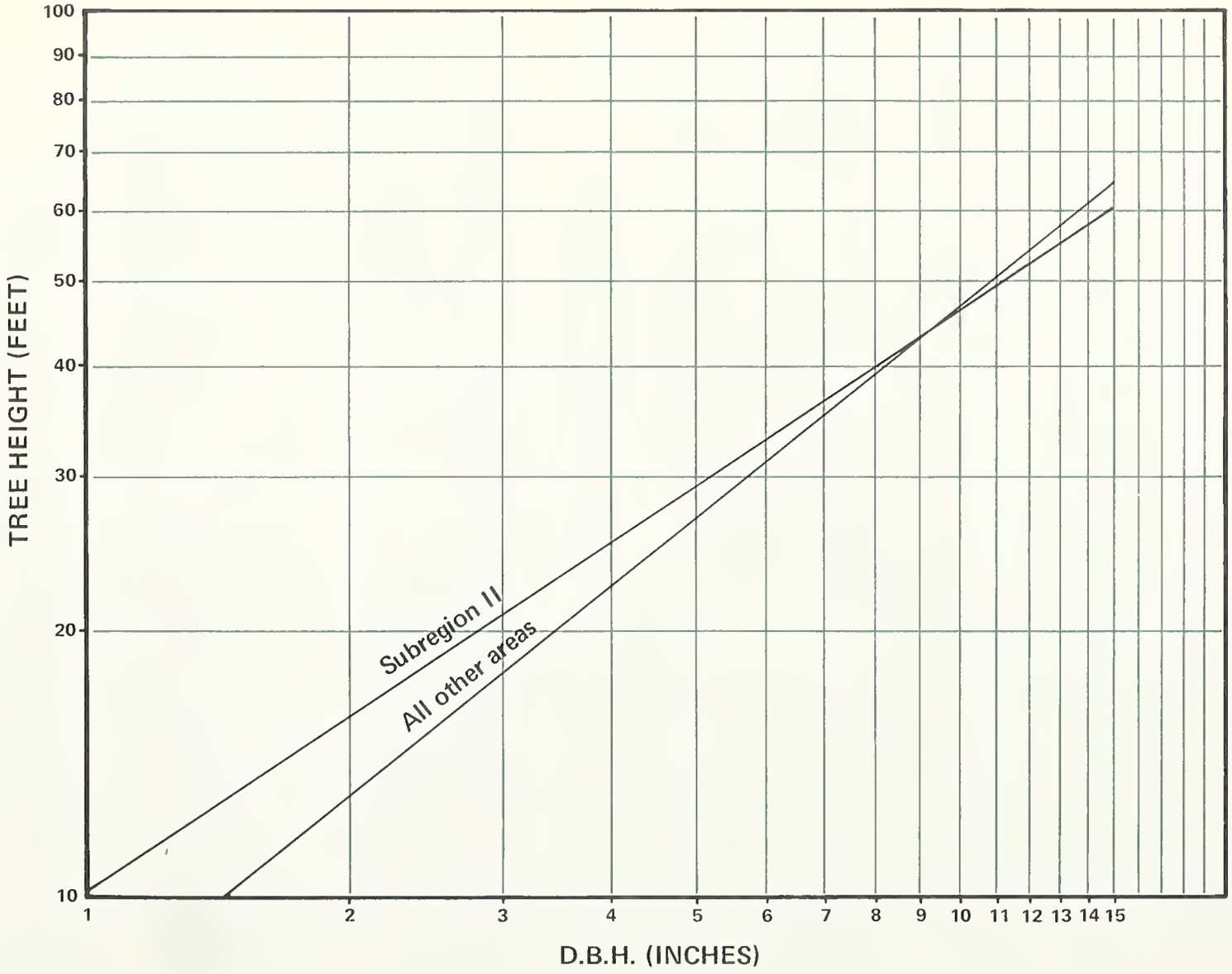


Figure 10.--Regressions of height on diameter for site trees less than 70 years old in subregion II, and in all other areas.

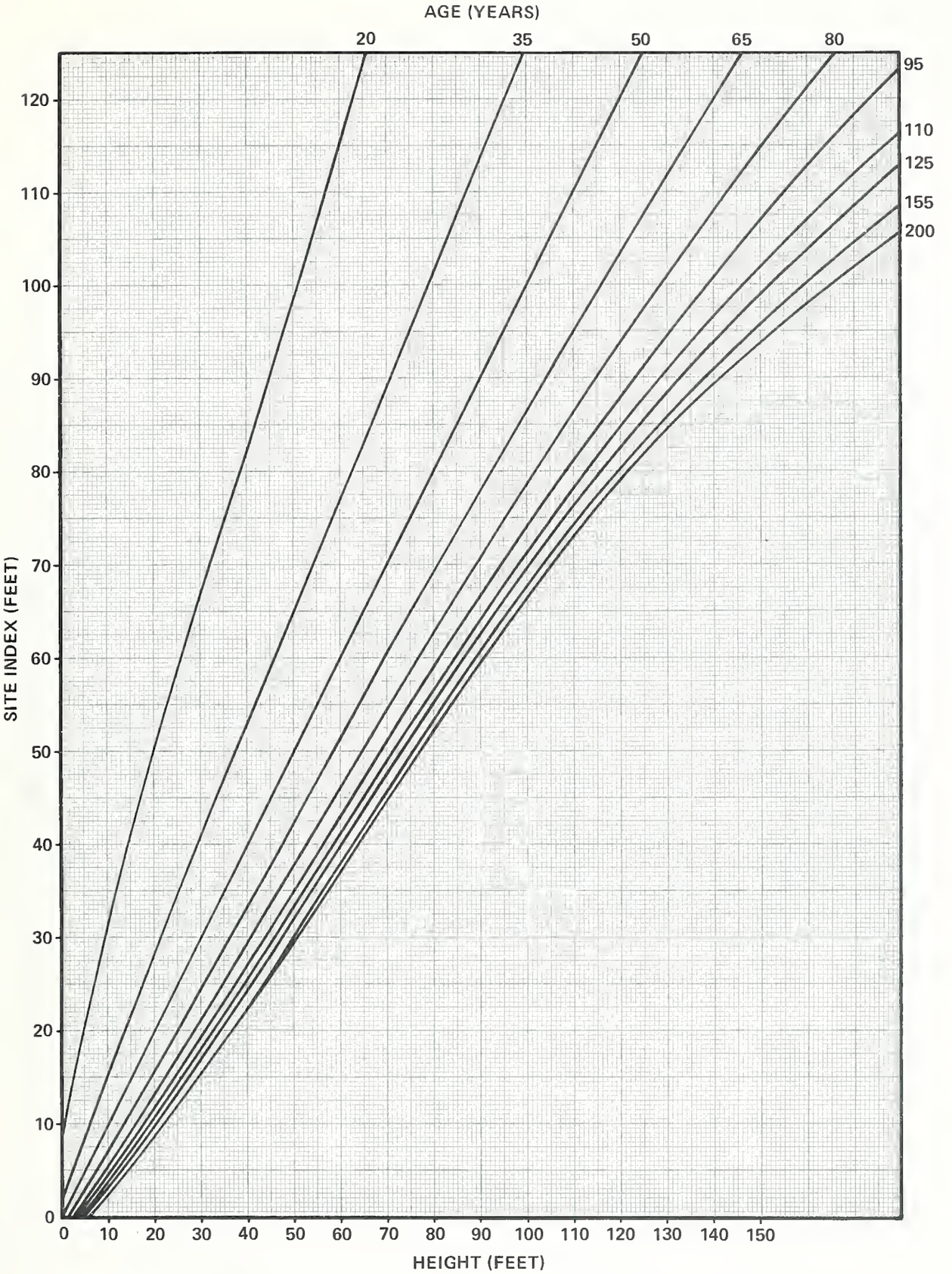


Figure 11.--The relationship between site index and height for several different ages.

age between 20 and 200, inclusive, resulted in 181 sets of coefficients. Plotted over age, these coefficients showed very definite trends, which could be described[6] by the equations:

$$\hat{b}_0 = c_1(A^{-\frac{1}{2}} - 50^{-\frac{1}{2}}) + c_2(A^{-2.5} - 50^{-2.5}) + c_3(A^{-3.5} - 50^{-3.5}) + c_4(A^4 - 50^4) + c_5(A^5 - 50^5) \quad (11)$$

$$\hat{b}_1 - 1 = k_1(A^{-1.5} - 50^{-1.5}) + k_2(A^{-4} - 50^{-4}) + k_3(A^{-5} - 50^{-5}) + k_4(A^4 - 50^4) \quad (12)$$

$$\hat{b}_2 = p_1(A^{-2} - 50^{-2}) + p_2(A^{-5} - 50^{-5}) + p_3(A - 50) \quad (13)$$

$$\hat{b}_3 = q_1(A^{-1} - 50^{-1}) + q_2(A^{-5} - 50^{-5}) + q_3(A^{1.5} - 50^{1.5}) + q_4(A^4 - 50^4) \quad (14)$$

$$\hat{b}_4 = r_1(A^{-\frac{1}{2}} - 50^{-\frac{1}{2}}) + r_2(A^{4.5} - 50^{4.5}) \quad (15)$$

If these equations are conditioned to pass through the origin as shown, the necessary coefficient values at age 50 are maintained. Substituting the equations for the b_i coefficients into equation (10) and adding tree height, an equation is obtained that expresses site index as a function of age and height. The final site index approximating equation is:

$$S = H + \sum_{i=1}^{18} b_i X_i, \text{ where} \quad (16)$$

S = site index in feet of total height at a total tree age of 50 years.

H = total height of a sample tree.

A = total age of a sample tree.

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[6] Satisfactory expressions for the coefficients were derived from regression analysis. In fitting the necessary regressions, all variables that contained a negative integer power of age were scaled to avoid subtraction of two very small numbers. If a variable was A^{-k}, for example, it would have been input to regression as $[(10^{2k}A^{-k}) - 2^k]$. The subtraction of 2^k makes the value of the variable equal to zero at age 50. All variables in the equation were so treated as to equal zero at age 50; therefore at age 50--the index base age--site index equals height.

$b_1 = 0.40984664E+02$ [7]	$X_1 = (A^{-\frac{1}{2}} - 50^{-\frac{1}{2}})$
$b_2 = 0.45211527E+04$	$X_2 = (A^{-2.5} - 50^{-2.5})$
$b_3 = 0.12305938E+06$	$X_3 = (A^{-3.5} - 50^{-3.5})$
$b_4 = -0.53328682E-08$	$X_4 = (A^4 - 50^4)$
$b_5 = 0.37808033E-10$	$X_5 = (A^5 - 50^5)$
$b_6 = 0.21664152E+03$	$X_6 = H(A^{-1.5} - 50^{-1.5})$
$b_7 = -0.15812149E+06$	$X_7 = H(A^{-4} - 50^{-4})$
$b_8 = 0.18940308E+07$	$X_8 = H(A^{-5} - 50^{-5})$
$b_9 = -0.10230592E-09$	$X_9 = H(A^4 - 50^4)$
$b_{10} = -0.60686119E+01$	$X_{10} = H^2(A^{-2} - 50^{-2})$
$b_{11} = -0.25351090E+05$	$X_{11} = H^2(A^{-5} - 50^{-5})$
$b_{12} = 0.33512858E-04$	$X_{12} = H^2(A - 50)$
$b_{13} = 0.17024711E-02$	$X_{13} = H^3(A^{-1} - 50^{-1})$
$b_{14} = 0.39836720E+03$	$X_{14} = H^3(A^{-5} - 50^{-5})$
$b_{15} = -0.88665409E-08$	$X_{15} = H^3(A^{1.5} - 50^{1.5})$
$b_{16} = 0.40019102E-14$	$X_{16} = H^3(A^4 - 50^4)$
$b_{17} = -0.46929245E-08$	$X_{17} = H^5(A^{-\frac{1}{2}} - 50^{-\frac{1}{2}})$
$b_{18} = -0.16640659E-20$	$X_{18} = H^5(A^{4.5} - 50^{4.5})$

The standard error of estimate and coefficient of multiple determination for this expression are, respectively,

$S_{yx} = 0.238673$ foot of site index

$R^2 = 0.999936$

---

[7] Here the letter E followed by a sign and two digits means that the number preceding the E is to be multiplied by 10 raised to the power of the two digits. That is, nE+02 means n X 10^2; nE-01 means n X 10^{-1}, where n is the decimal fraction preceding the E.

If a shorter equation, with less precision, is desired, the following may be used:

$$S = H + \sum_{i=1}^{5} c_i Y_i, \text{ where} \qquad (17)$$

$c_1 = 0.10108708E+03$ $\quad Y_1 = (A^{-\frac{1}{2}} - 50^{-\frac{1}{2}})$

$c_2 = 0.12204763E+03$ $\quad Y_2 = H(A^{-1.5} - 50^{-1.5})$

$c_3 = 0.14082397E-12$ $\quad Y_3 = H(A^{5} - 50^{5})$

$c_4 = -0.13140717E+05$ $\quad Y_4 = H^2(A^{-5} - 50^{-5})$

$c_5 = 0.13492191E-04$ $\quad Y_5 = H^5(A^{-3} - 50^{-3})$

For this equation $S_{yx} = 0.904479$ and $R^2 = 0.999080$.

These equations are valid for ages from 20 to 200 years and for site indices from 10 to 110. The site index of trees older than 200 years should be estimated as if age were 200.

HOW TO USE THIS MATERIAL

To estimate the average site index for a stand of inland Douglas-fir, measure total age and total height of trees that are in the dominant portion of the stand. Site trees also should appear to have been in the dominant stand throughout their lives. The number of sample trees necessary to give any stated precision of estimate will depend on the variance in site index between trees in the stand. This is influenced by the size of the stand and its heterogeneity.

Estimate a site index value for each sample tree by using any one of the following:

1. The curves on the tearout page.

2. Figure 11 on page 19.

3. Table 3 on page 14.

4. Either equation 16 on page 20 or equation 17 on this page.

Compute the average of the site index estimates for each sample tree. This average will be the best estimate of site index for the area sampled.

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RECENT FOREST SURVEY PUBLICATIONS

Title	Number	Date
Timber Resources of Idaho	Forest Survey Release 3	1962
The Forest Resources of Western Montana	Resource Bulletin INT-1	1963
*The Forests of Wyoming	Resource Bulletin INT-2	1963
*The Forest Resources of Colorado	Resource Bulletin INT-3	1964
Output of Timber Products in Montana, 1962	Research Paper INT-11	1964
Forest Products Output in Utah and Nevada, 1962	Research Paper INT-12	1964
Output of Timber Products in Idaho, 1962	Research Paper INT-13	1964
*Timber Products Output in Colorado, Wyoming, and Western South Dakota, 1962	Research Paper INT-14	1964
*Timber Products Output in Arizona and New Mexico, 1962	Research Paper INT-15	1964
Forests in Utah	Resource Bulletin INT-4	1965
Site Index Curves for Engelmann Spruce in the Northern and Central Rocky Mountains	Research Note INT-42	1966
*New Mexico's Forest Resource	Resource Bulletin INT-5	1966
*Arizona's Forests	Resource Bulletin INT-6	1966
Timber and Timber Industries in the Rocky Mountain States	Resource Bulletin INT-7	1967

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